CHAPTER ONE

Indices Exponential Equations And Application Of Algebra

Indices:

1. $a^n \times a^m = a^{n+m}$
   This is the first law of indices.
2. $a^n \div a^m = a^{n-m}$
   This is the second law of indices.

   Examples:
1. $2^3 \times 2^4 = 2^{3+4} = 2^7$
2. $2^6 \times 2^4 = 2^{6+4} = 2^{10}$
3. $3^4 \times 3 = 3^4 \times 3^1 = 3^{4+1} = 3^5$
4. $3^6 \times 3^2 = 3^{6+2} = 3^8$
5. $2^3 \times 2^4 \times 2^2 = 2^{3+4+2} = 2^9$
6. $4^3 \times 4 \times 4^2 = 4^3 \times 4^1 \times 4^2 = 4^{3+1+2} = 4^6$
7. $2^6 \div 2^4 = 2^{6-4} = 2^2$
8. $3^8 \div 3^2 = 3^{8-2} = 3^6$

Simplify the following:

1. $\frac{2^3 \times 2^6}{2^4}$

Soln.

$\frac{2^3 \times 2^6}{2^4} = \frac{2^{3+6}}{2^4} = \frac{2^9}{2^4} = 2^9 \div 2^4 = 2^{9-4} = 2^5$.

2. $\frac{3^4 \times 3^6}{3} = \frac{3^4 \times 3^6}{3^1} = \frac{3^{4+6}}{3^1} = \frac{3^{10}}{3^1} = 3^{10-1} = 3^9$.
3. $\frac{2^4 \times 2 \times 2^3}{2^6 \times 2^2} = \frac{2^{4+1+3}}{2^{6+2}} = \frac{2^8}{2^8} = 2^{8-8} = 2^0 = 1$

N/B: Any number raised to the power zero = 1.

4. $9^{a^2} \times 3^{a^{-6}} = 9 \times 3 \times a^2 \times a^{a-6} = 27 \times a^{2+-6} = 27\times a^{2-6}$
   $= 27a^{-4}$

5. $a^{-4} \times a^{-2} = a^{-4+-2} = a^{-4-2} = a^{-6}$
6. $3a^{-4} \times 2a^{-3} = 3 \times 2\, a^{-4} \times a^{-3} = 6 \times a^{-4+-3}$
$= 6a^{-4-3} = 6a^{-7}$

N/B: $\frac{1}{a^2} = 1 \times a^{-2} = a^{-2}$

2. $\frac{1}{a^{-2}} = 1 \times a^2 = a^2$

3. $\frac{1}{3^2} = 1 \times 3^{-2} = 3^{-2}$
4. $\frac{1}{3^{-2}} = 1 \times 3^2 = 3^2$
5. $\frac{9}{3a^{-2}} = \frac{9}{3} \times a^2 = 3 \times a^2 = 3a^2$
6. $\frac{9}{3a^2} = \frac{9}{3} \times a^{-2} = 3a^{-2}$
7. $\frac{4}{2a^2} = \frac{4}{2} \times a^{-2} = 2 \times a^{-2} = 2a^{-2}$
8. $\frac{4}{2a^{-2}} = \frac{4}{2} \times a^2 = 2 \times a^2 = 2a^2$
9. $\frac{4a^{-6} \times 2a^2}{2a^{-3}} = \frac{4 \times 2 \times a^{-6} \times a^2}{2 \times a^{-3}}$
$= \frac{8 \times a^{-4}}{2 \times a^{-3}} = 4 \times a^{-4} \times a^3$
$= 4 \times a^{-4+3} = 4a^{-1}$
10. $\frac{2a^{-2} \times 6a^4}{3a} = \frac{2 \times 6 \times a^{-2} \times a^4}{3 \times a}$
$= \frac{12a^{-2+4}}{3 \times a^1} = \frac{4a^2}{a^1} = 4a^2 \times a^{-1}$
$= 4a^{2-1}$
$= 4a^1 = 4a$
11. $3a^2 b \times 4a^3 b^4 = 3 \times 4 \times a^2 \times a^3 \times b^1 \times b^4$
$= 12a^5 b^5$
12. $3a^{-3} b^2 \times 5a^{-2} b^{-4} = 3 \times 5 \times a^{-3} \times a^{-2} \times b^2 \times b^{-4}$

$= 15a^{-5}\ b^{-2}$

13. $\dfrac{3a^2\ b \times 6a^3\ b^4}{2ab^2} = \dfrac{3 \times 6 \times a^2 \times a^3 \times b \times b^4}{2 \times a \times b^2}$

$= \dfrac{18a^5b^5}{2 \times a \times b^2} = 9 \times a^5 \times a^{-1} \times b^5 \times b^{-2}$

$= 9a^4\ b^3$

Exponential Equations:

N/B: 1. $4 = 2^2$

3. $8 = 2 \times 2 \times 2 = 2^3$
4. $32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$
5. $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6$
6. $25 = 5 \times 5 = 5^2$
7. $125 = 5 \times 5 \times 5 = 5^3$
8. $16 = 4 \times 4 = 4^2$
9. $64 = 4 \times 4 \times 4 = 4^3$
10. $9 = 3 \times 3 = 3^2$
11. $27 = 3 \times 3 \times 3 = 3^3$
12. $81 = 9 \times 9 = 9^2$
13. $81 = 3 \times 3 \times 3 \times 3 = 3^4$

Q1. If $2^x = 8$, find x.

Soln.

Since $8 = 2 \times 2 \times 2 = 2^3$, then $2^x = 8 \Rightarrow 2^x = 2^3 \Rightarrow x = 3$.

Q2. Given that $2^{x+1} = 16$, find x.

Soln.

$16 = 2 \times 2 \times 2 \times 2 = 2^4$

$\therefore 2^{x+1} = 16 \Rightarrow 2^{x+1} = 2^4$

$\Rightarrow x + 1 = 4, \Rightarrow x = 4 - 1 = 3$.

Q3. If $4^{2x-1} = 64$, find x.

Soln.

$64 = 4 \times 4 \times 4 = 4^3$

$\therefore 4^{2x-1} = 64$

$\Rightarrow 4^{2x-1} = 4^3$

$\Rightarrow 2x - 1 = 3,$

$\Rightarrow 2x = 3 + 1 = 4.$

$\therefore x = {}^4/_2 = 2.$

Q4. Given that $5^{2(n-1)} = 125$, find n.

Soln.

$125 = 5 \times 5 \times 5 = 5^3$

$\therefore 5^{2(n-1)} = 125$

$\Rightarrow 5^{2(n-1)} = 5^3$

$\Rightarrow 2(n - 1) = 3, \Rightarrow 2n - 2 = 3,$

$\therefore 2n = 3 + 2 \Rightarrow 2n = 5,$

$\therefore n = {}^5/_2 = 2.5.$

Q5. Given that $2^{x+1} = 8^x$, find x.

Soln.

$2^{x+1} = 8^x$, but $8 = 2^3$

$\Rightarrow 2^{x+1} = 2^{3x}$

$\Rightarrow x + 1 = 3x, \Rightarrow 1 = 3x - x$

$\Rightarrow 1 = 2x,$

$\Rightarrow x = {}^1/_2$ or $x = 0.5.$

Q6. If $3^{2(x-1)} = 27^x$, determine the value of x.

Soln.

$27 = 3^3$

$\therefore 3^{2(x-1)} = 27^x$

$\Rightarrow 3^{2(x-1)} = 3^{3x}$

$\Rightarrow 2(x - 1) = 3x, \Rightarrow 2x - 2 = 3x,$

$\Rightarrow 2x - 3x = 2,$

$\Rightarrow -x = 2 \Rightarrow x = -2.$

Q7. Given that $2^{x-1} = 16^{x+1}$, find the value of x.

Soln.

$16 = 2^4$

$\therefore 2^{x-1} = 16^{x+1}$

$\Rightarrow 2^{x-1} = 2^{4(x+1)}$

$\Rightarrow x - 1 = 4(x + 1),$

$\Rightarrow x - 1 = 4x + 4,$

$\Rightarrow x - 4x = 4 + 1,$

$\Rightarrow -3x = 5$

$\Rightarrow x = {}^{-5}/_{3} = -1.6.$

N/B: If there is a plus or a minus sign between a number and a letter, they must be placed within a bracket before a number can be used to multiply them.

Q8. If $3^n = 81^{n-1}$, find the value of n.

Soln.

$81 = 3^4$

$\therefore 3^n = 81^{n-1} \Rightarrow 3^n = 3^{4(n-1)}$

$\Rightarrow n = 4(n - 1),$

$\Rightarrow n = 4n - 4$

$\Rightarrow n + 4 = 4n,$

$\Rightarrow 4 = 4n - n \Rightarrow 4 = 3n,$

$\Rightarrow n = {}^{4}/_{3} = 1.3.$

Q9. If $3^{2(x-1)} = 27^{x+2}$, find the value of x.

Soln.

$27 = 3^3$

$\therefore 3^{2(x-1)} = 27^{x+2}$

$\Rightarrow 3^{2(x-1)} = 3^{3(x+2)}$

$\Rightarrow 2(x - 1) = 3(x + 2),$

$\Rightarrow 2x - 2 = 3x + 6,$

$\Rightarrow -2 = 3x + 6 - 2x,$

$\Rightarrow -2 - 6 = 3x - 2x$

$\Rightarrow -8 = x,$

$\therefore x = -8.$

Q10. Given that $(^1/_2)^{n+1} = {}^1/_4$, determine the value of n.

Soln.

$^1/_4 = (^1/_2)^2$

$\therefore (^1/_2)^{n+1} = ¼$

$\Rightarrow (^1/_2)^{n+1} = (^1/_2)^2$

$\Rightarrow n + 1 = 2, \Rightarrow n = 2 – 1$

$\Rightarrow n = 1.$